TE 855

Teaching School Mathematics

Fall 2010

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Don't hesitate to contact me to arrange a one-on-one, face-to-face meeting.
Or we can just chat via Skype or on the phone about any concerns/questions you may have.

Required Texts:

Lampert, M. (2001) *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

Available online: National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics* at
http://standards.nctm.org. Although the site is designed for use by NCTM members, you will be able to sign up to use it for free for 120
days. Other assigned readings will be posted on our class wiki.

Course Purpose and Goals:

Although reasoning and proving are central to the discipline of mathematics, school mathematics has often been taught as a collection of rules and procedures that can be memorized and used to solve the teacher's and the textbook's math problems. A steady diet of rules and procedures not only serves to turn off students from mathematics but also deprives them from experiencing the intellectual work entailed in doing mathematics—but perhaps most importantly it undermines the expectation that mathematics is supposed to make sense.

Consider the "How Old is The Shepherd?" problem:

There are 125 sheep and 5 dogs in a flock. How old is the shepherd?
This problem was given to students across grades K-4 as part of a national assessment project. Surprisingly, students in the younger grades were able to recognize the question about the shepherd's

age as not answerable (with the given information). In contrast, most students in the older grades managed to produce a numerical answer after doing some computations with the given numbers. Here's an example from a third grader:

125+5=130 ... this is too big, and 125-5=120 is still too big ... while 125/5=25. That works! I think the shepherd is 25 years old.

This example is one of many reports of how students of mathematics (of all ages) end up with a poor understanding of the subject but it is also an example of how school mathematics is failing to prepare students to become literate and active citizens in a democratic society. This is especially concerning because much of the research suggests that as students progress through the grades, they come to stop asking the most basic of math questions --- does this make sense? Why does this work? … and so on.

In recent years, however, schools and teachers are being urged to move away from memorizing procedures and instead "teach for conceptual understanding." In many classrooms this has meant focusing new attention on young people's mathematical reasoning.

In Principles and Standards for School Mathematics, the National Council of Teachers of Mathematics (NCTM) identifies reasoning and proof as one of the most important strands of a good mathematics curriculum – whether in kindergarten, in fifth grade, in middle school, or in high school. This does not mean that we should teach reasoning and proof instead of two-digit multiplication, subtraction of decimals, factoring of polynomials, or calculus; it means that we need to teach all math content in ways that helps students to see mathematics as reasonable and themselves as people who can reason their way into new mathematical ideas. In other words, we want to teach children and adolescents <u>how to reason mathematically</u> and we want to teach them that <u>math is reasonable</u>. We want them to know that it makes sense to ask "why does that work?" when they learn to do something new in math class and to expect that with the help of their teacher and their classmates they will be able to reason their way to a sensible answer.

The focus of TE-855, Teaching School Mathematics, is on mathematical reasoning for three reasons. First, mathematical reasoning is fundamental to the development of robust mathematical understandings. Indeed, all conceptual understanding of math is based in mathematical reasoning, in experiences and conversations that allow us to see how the mathematics we are learning and using relates logically to other mathematics that we already understand. It would be difficult to say where mathematical reasoning stops and mathematical understanding begins. Second, mathematical reasoning is the foundation for the other cross-cutting standards that the Principles and Standards for School Mathematics identifies as important to mathematics teaching at all levels, namely: communication, connections, problem solving, and representation. For example, as John Van de Walle (2003) points out in his book on mathematics teaching, students construct rich understandings of a new mathematical idea by making connections between the new idea and the network of mathematical understandings they already have; "Rote knowledge [knowledge without reasoning] will almost never contribute to a useful network of ideas." Third, as we have said above, mathematical reasoning is at the core of "doing mathematics." It is central not only to the work of professional mathematicians, designers, and engineers, but also to the efforts of ordinary adults to use math for their own purposes. When, for example, the person in front of you at Home Depot has to decide how much plywood they need, but cannot remember how to divide by a mixed number, they must reason from what they do remember to

what they need to figure out.

In TE-855 we will consider what mathematical reasoning might look like at different grade levels (including kindergarten!) and what it takes to teach students to reason mathematically and to build on one another's mathematical reasoning. Of course we will not be able to cover what mathematical reasoning looks like in every corner of the K-12 curriculum, but we will range widely across the content of school mathematics and will draw on the experience and challenges that each of you brings with you to our weekly meetings.

Our class is likely diverse: some are teaching, or have taught, elementary school math while others have experience teaching algebra, geometry, or trigonometry in middle school or high school; some are teaching math now while others are currently full-time students. In order to speak as directly as possible to the concerns of as many of you as possible, we will read from a diverse set of authors that are teachers of elementary and secondary school mathematics.

Many of you took university courses that helped (we hope) to prepare you to teach mathematics. At that time most of you did not have your own classrooms or sole responsibility for the mathematics learning of children or adolescents (although you may have been interning or student teaching as you took some of those courses). Now, however, many of you are or have been teachers of elementary or secondary mathematics. You will bring cares and concerns from your teaching into our class; your responsibilities, struggles, and misgivings (past and present) will shape the meaning you draw from the readings, your participation in discussions, and the choices you make in designing your classroom investigations and your research project. The particulars of your teaching are not a distraction from the course: they are a resource to all of us and we welcome them into our discussion.

We will attend to six big questions across the term (and many smaller ones from week to week):
- What are our experiences with mathematical reasoning? Where do we come from?
- How do we know mathematical reasoning when we see/hear it?
- If our goal is to help students learn to reason mathematically, to see school math as reasonable, what makes a good task?
- Are there particular pedagogical practices, and particular sorts of classroom culture, that help K-12 students learn to reason mathematically?
- How does teaching mathematical reasoning fit with our other priorities, obligations, and commitments?
- How do we assess our success in teaching mathematical reasoning?

Course Activities:

1. *Participation in Discussion Boards (10%)*
It is absolutely essential that you participate actively and regularly in the discussions on the wiki.

A Word on Participation …
It is hard for some students to participate enthusiastically in discussions about math, because mathematics is not a realm in which they feel confident. This fact influences both the substance of our work and the way we talk: We have to think extra hard about how to create mathematics learning

communities in school classrooms because we know how hard it is to make everyone feel that their mathematical ideas are valuable; and in our own online class we have to work hard on listening to one another and on making TE-855 a safe place to offer "wrong" or unpopular ideas and to ask questions that may seem dumb to the person who asks but will probably express other people's confusions and uncertainties too. We also, paradoxically, have to work to make our class a safe place to be "right": Sometimes students who enjoy mathematics feel reluctant to share their ideas because they worry that others will see them as know-it-alls. Our most basic expectation is that we will listen to one another with interest and respect and to show that respect and interest by asking for clarification when we aren't sure we understand an idea, by raising questions when we aren't sure we agree, and by explaining why when we do agree. But it is also very important to read the readings faithfully, thinking hard about them as you read, and to "speak up" even when you feel a little shy about doing so.

2. *Weekly Reflective Writing (50%)*

Each week I will give an assignment for reflective writing about an assigned reading. Most of these assignments will relate to assigned readings; some of them will ask you to design a bit of curriculum or to try something in a classroom (do not worry: I know that some of you do not have your own math classroom so we can together figure out another way to work on such assignments). Here is an example of the kind of weekly assignment:

Read: Ball, D. & Bass, H. (2003). Making mathematics reasonable in school. In the *Research Companion to the Principles and Standards for School Mathematics.* Reston, VA: NCTM.

Before reading …
Some third graders in this article say that an odd number plus an odd number equals an even number because they have tried a lot of examples and they always got an even number. Others in the class say that you can't know that that's always true because numbers go on forever. What do you think about the mathematical reasoning in these claims and why?

After reading …
Ball and Bass have some things to say about making mathematics reasonable in schools.
• What did they say that you found particularly convincing and what raised questions for you?
• In what ways did what you read connect with what you do as a teacher of mathematics and/or what you experienced as a school student?

3. Research Project and updates (40%)
You will choose a question or issue that connects in some way to the teaching of mathematical reasoning or proof and is of particular interest to you.

1. If you are currently teaching math, the research project is an opportunity for you to try some things in your classroom that you think might help you to "make math reasonable in school" and to learn from your efforts. You will design an intervention – something you want to try – and with my help and the help of your classmates you will design an approach to learning from your efforts. This work with your own students will be the heart of your research project. However, as a part of planning your investigation, you will identify a question you will be trying to answer – or at least learn more about – and read as much as you can about this question.

2. A few of you do not have your own classroom or are not teaching math. Doing some library research connected to a question about making math reasonable in school is one option for your research project. However, because I have found that students seem to learn more by pursuing their questions with actual K – 12 students, I will ask you to try to find a classroom or group of students with whom you can work. If that is not feasible, you and I can think together about whether there is a way for you to use other existing data (transcripts or video that I am aware of, for example) to get further insight into your question. And,of course, if that can't work we will find ways for you to learn more about your question from library and internet research. Where there's a will, there's a way!

You will begin by trying to formulate a question you want to get smarter about. You will look for some readings – including readings for the course – and by thinking about what they tell you about your question. You will think about how you might investigate this question in your classroom. What might you plan to do? How might you analyze your data? I will give you help and suggestions and I will also connect you, through small group discussion boards, with other students in the course who share some of your interests so you can help them and they can help you.

In order to make sure that you have time and resources to formulate and explore a question that matters to you, I have scheduled four research updates across the semester. On September 30 you will write to me about questions that have arisen for you over the previous month and ideas you have for your investigations; On October 14 you will update me on the ways your question is evolving, how you think you might use your classroom (or perhaps someone else's) to explore a part of that question, and some preliminary ideas about your bibliography; On October 28 to November 4 you will give and get help on finding more good readings; and on November 25 you will share some "data" and some of what you think you are learning with your on-line partners, and get feedback on ways to think about your data. The final draft of the project is due to me on December 9.

Grading:

Final grades will be assigned as follows:
Out of 100 points 4.0 scale equivalent
94 - 100 4.0
88-93 3.5
82-87 3.0
76-81 2.5
70-75 2.0
Each assignment will be assessed using the following criteria:

1. Thoughtfulness – Does the response draw on, but go beyond, reporting and synthesizing the readings?
2. Responsiveness – Does the response demonstrate clear understanding of the topic addressed?
3. Effectiveness – Does the argument follow a logical development? Is the evidence provided adequate to support the argument? Does the author make clear how referenced works support

the argument? If appropriate, does the author anticipate and address counter-arguments? Are other perspectives considered?

4. Clarity of communication – Is the response comprehensible? Does the author's use of linguistic conventions (grammar, syntax, organization) and of language allow the reader to follow the argument?

I aim to return feedback on all written work within 2 weeks after it has been turned in, if not sooner.

Week 1-2: What are our experiences with mathematical reasoning?

Introductions to each other and the course. What questions or experiences come to mind when we think of 'reasoning and proof'? What are some mathematical claims we have accepted to be true yet we are not really convinced or have a real proof that these are so or why these are true. What do we mean and understand by mathematical reasoning? Examine and discuss in the context of some math work and readings:

1. Lampert Chpts 1&2
2. Chazan, D. (2000). Beyond Formulas in mathematics and teaching: Dynamics of the high school algebra classroom. Chap. 1: 'My Algebra teaching autobiography'
3. NCTM. (2000). Principles and Standards for School Mathematics: Reasoning and Proof in all grade bands PreK-2; Grades 3-5; Grades 6-8; and Grades 9-12.
4. **Research Project Assignment:** Brainstorming and brewing your own drafty definition about mathematical reasoning in relation to a piece of mathematics you're puzzling over.

Week 3-5: How do we know mathematical reasoning when we see/hear it?

Considering evidence that students are (and not) doing mathematical reasoning and how they were supported (or not) in this mathematical activity.

1. Ball & Bass reading
2. Yackel & Hanna reading
3. Lampert Ch. 6: "Teaching while students work independently"
4. Readings from Carpenter, T., Franke, M., & Levi, L. (2003). Thinking mathematically: Integrating arithmetic and algebra in elementary school. Portmouth, NH: Heinemann.
5. **Research Project Assignment:** Formulate questions for possible investigations for course research project

Week 6-8: The nature of mathematical tasks and how they promote or discourage students' mathematical reasoning

We will consider the nature and quality of mathematical tasks that support students' mathematical reasoning. We'll study a few classification schemes and develop criteria for judging the quality of mathematical tasks—in particular whether they can promote students' mathematical reasoning.

1. Lampert Ch. 5 --- Teaching while preparing for a lesson

2. Chazan Ch. 2 --- Curricular engagement and personal trajectories
3. Smith, M.S., Stein, M.K., Arbaugh, F., Brown, C.A., & Mossgrove, J. (2004). Characterizing the cognitive demands of mathematical tasks: A task-sorting activity. In *Professional Development Guidebook for Perspectives on the Teaching of Mathematics*. G.W. Bright & R.N. Rubenstein (Eds). p. 45-72. National Council of Teachers of Mathematics: Reston, VA.
4. **Research Project Assignment:** Find and abstract 2 references related to your research project question(s).

Week 9-11: Are there particular pedagogical practices that help K-12 students learn to reason mathematically?

Good tasks and good intentions are not enough to support students' mathematical reasoning. Although all mathematics instruction starts with a good task, teachers' instructional decisions play significant roles in encouraging students to see mathematics as reasonable, and to persist in making sense when they get stuck. We will examine ways in which teachers organize mathematics instruction and how they use instructional strategies to promote their students' reasoning.

1. Lampert, Ch. 4: "Teaching to establish a classroom culture"
2. Lampert, Ch. 10: "Teaching students to be people who study in school."
3. Chazan Ch. 4: 'Developing conversations in the mathematics classroom'
4. Readings from: Herbel-Eisenmann, B. & Cirillo, M. (2009). Promoting purposeful discourse: Teacher research in secondary math classrooms. Reston, VA: NCTM.
5. **Research Project Assignment:** Share an artifact from your research project with classmates to do a collaborative analysis of it.

Week 12-13: How does teaching mathematical reasoning fit with our other priorities, obligations, and commitments?

Teaching mathematics does not happen in a vacuum, it is intertwined with multiple and sometimes competing goals and commitments, some of which are our own, some which are policies and mandates. In these final weeks we'll discuss ways in which we can make mathematical reasoning a reasonable and feasible goal in our individual school contexts.

1. Lampert Ch. 8: "Teaching to connect content across lessons,"
2. Lampert Ch. 9: "Teaching to cover the curriculum."

3. **Research Project Assignment:** Sharing insights from research project – Powerpoint to share with classmates.

Week 14-15: How do we assess our success in teaching mathematical reasoning? Getting back to the question of where are we coming from and charting where we are heading!

1. Senk & Thompson reading
2. Re-imagining mathematics teaching …